

Leaving where I grew up to study in the US is the boldest decision I have ever made in my life, and I never looked back. The experience was multi-faceted; I learned the local lingo, got accustomed to a new place, and met a lot of new people. Throughout this whole process, I was sensitive to the cultural differences, and I enjoyed learning from my peers while also sharing my own experiences and perspectives that I had gained from my time in Singapore. It was a nerve-wracking yet empowering experience. To me, studying abroad is another opportunity for me to broaden my global perspective, both on a personal and academic level. I have never been to the UK, and I am extremely excited to immerse myself in a new environment.

My educational experience in Caltech has been so different from my experience back home, and this has made me even more curious and eager to study abroad in the UK. The aspect of the European educational system that intrigues me most is the emphasis placed on self-review. This philosophy is especially appropriate in the context of mathematics, since the best way to internalise mathematical concepts is through independent problem solving. I am eager to experience the change of pace at Cambridge and Edinburgh, where greater importance is placed on intimate guidance. I am particularly excited about the unique supervision system in Cambridge where students work closely with faculty members and graduate students, something that Caltech does not meet to the same degree. In addition, I would be able to take courses that are not offered at Caltech, such as *Quantum Information and Computation* and *Riemann Surfaces* at Cambridge, and *Blockchains and Distributed Ledgers* at Edinburgh. After my recent SURF on the group-theoretic approach to matrix multiplication, I also realised that I want to pursue academia, and studying abroad will give me a taste of what it might be like to attend graduate school in the UK.

To me, the most meaningful aspect of studying abroad is the time spent outside the classroom. Since I spent most of my life in Singapore, a relatively new country, I have always yearned to visit more historical places. If I study abroad, I plan on making frequent weekend trips to visit cultural places such as Edinburgh Castle and the Fitzwilliam Museum. I also plan on getting involved in clubs such as the Global Kitchen Society and SocieTea at Edinburgh, and the Bridge Club and Nature Society at Cambridge. I am also extremely excited to meet new people and make life-long connections. I have been in STEM-based environments for almost a decade, so I am looking forward to meeting students studying a broader range of subjects with more diverse backgrounds.

Studying abroad will complete my undergraduate experience. I am so excited to step out of my comfort zone and immerse myself in a new environment, take courses taught by world-renowned professors, and make new friends. I am convinced that studying abroad will broaden my perspectives and allow me to grow as a person.

In my junior year of Caltech, I am focusing on completing biennially offered theoretical computer science electives, a three-term sequence in geometry and topology, a project sequence in machine learning, and research in combinatorics. Therefore, in my senior year, I am hoping to take courses to complement my interests in algebra, analysis, and theoretical computer science, and I have planned my schedules for studying abroad with this in mind.

Choices #1 and #2: Cambridge University

As someone who was accepted to both Cambridge and Caltech for my undergraduate studies, I was torn between the two. My decision to ultimately attend Caltech was in part because I would be able to study abroad in Cambridge! I am most excited about studying abroad at Cambridge's unique supervision system. As part of my learning style, I spend a substantial amount of time working on problem sets on my own, before working with friends. I love the collaboration that is heavily encouraged in Caltech, but the frequent sets give me less time to work independently. The Cambridge system allows me to do this with more flexibility, since the exercise sheets are longer but less frequent. I have also come to appreciate close interactions with professors during my reading classes and research experiences, and I am looking forward to working with faculty from Cambridge during these supervisions. I am also excited to attend Math talks organised by Adams Society of St. John's college, and to meet esteemed professors at Cambridge.

Choice #1 - Lent term: I would prefer to study during Lent term. I will be able to take a course in quantum computation that is more accessible to math majors, and I am extremely excited this will be taught by Professor Richard Jozsa, one of the pioneers of the field. I will also be able to take a course in *Representation Theory*, a field that is important in the context of theoretical computer science. I can also take *Logic and Set Theory* taught by Professor Imre Leader, someone I have met before since he regularly visited my high school in Singapore. I had the opportunity to attend several talks that he gave and interact with him personally, and I am extremely excited to reconnect.

Choice #2 - Michaelmas Term: I am most excited about the course on *Riemann Surfaces*, which is a truly unique course that isn't offered at Caltech. I will also be taking *Probability and Measure*, since this course will allow me to pursue to Ma/ACM/IDS 144ab sequence in Caltech. Finally, I would love to take *Number Theory* and *Classical Dynamics*, since they complement my interdisciplinary interests in cryptography and physics.

Cambridge Proposed Course List – Lent Term

Total CIT Units for term abroad: 36

Course by Correspondence/Units: 0

1. Class/Module title *Logic and Set Theory*

Tripes Mathematics

Part *IID*

Term (Michaelmas or Lent) *Lent*

Number of lectures *24*

Lecture times, if available *M, W, F 9am*

Caltech units *9*

Caltech evaluator Alexander Kechris

Type of Caltech credit (option, general, etc.) *Option Elective Credit*

State CIT equivalent course, if applicable *Ma 116b*

Course description (paste in paragraph)

The aim of this course is twofold: to provide you with an understanding of the logical underpinnings of the pure mathematics you have studied in the last two years, and to investigate to what extent, if any, the 'universe of sets' can be considered as a structure in its own right. As such, it has few formal prerequisites: some familiarity with naive set theory, as provided by the IA Numbers and Sets course, is helpful, but no previous knowledge of logic is assumed. On the other hand, the course has links to almost all of pure mathematics, and examples will be drawn from a wide range of subjects to illustrate the basic ideas. The course falls into three main parts. One part develops the notions of validity and provability in formal logic, culminating in the Completeness Theorem, which asserts that these two notions coincide. Another part is concerned with ordinals and cardinals: these are notions that generalise the ideas of size and counting to the infinite. The final part is an introduction to formal set theory, where one makes precise the idea of a 'universe of sets', and studies its structure. The book 'Notes on Logic and Set Theory' by P.T. Johnstone (C.U.P., 1987) covers most of the material of the course, and is suitable for preliminary reading.

2. Class/Module title *Representation Theory*

Tripes Mathematics

Part *IID*

Term (Michaelmas or Lent) *Lent*

Number of lectures *24*

Lecture times, if available *T, Th, S 9am*

Caltech units *9*

Caltech evaluator *Eric Rains*

Type of Caltech credit (option, general, etc.) *Option Elective Credit*

State CIT equivalent course, if applicable *Ma 145a*

Course description (paste in paragraph)

This course, suitable for pure and applied mathematicians, is an introduction to the basic theory of linear (matrix) actions of finite groups on vector spaces. The key notion we define is the character of a linear representation: this is a function on conjugacy classes of the group which determines the representation uniquely. Orthogonality relations between characters lead to a convenient and efficient calculus with representations, once the basic character table of the group has been computed. Later in the course 'finite' is replaced by 'compact' generalising the results with little extra effort. The Linear Algebra course is essential and Groups, Rings and Modules is helpful.

3. Class/Module title *Quantum Information and Computation*

Tripes Mathematics

Part *IIC*

Term (Michaelmas or Lent) *Lent*

Number of lectures *24*

Lecture times, if available *M, W, F 11am*

Caltech units *9*

Caltech evaluator *Thomas Vidick*

Type of Caltech credit (option, general, etc.) *CS Elective Credit*

State CIT equivalent course, if applicable *Ph/CS 219a*

Course description (paste in paragraph)

Quantum processes can provide enormous benefits for information processing, communication and security, offering novel features beyond the possibilities of standard (classical) paradigms. These benefits include (i) new kinds of algorithms (so-called quantum algorithms) providing an exponentially faster method for some computational tasks, (ii) new modes of communication such as quantum teleportation, and (iii) the possibility of unconditionally secure communication in quantum cryptography. Most of these exciting developments have occurred in just the past few decades and they underpin striking applications of quantum technologies that are currently being developed. This course will provide an introduction to these topics. No previous contact with the theory of computation or information will be assumed. IB Quantum Mechanics is essential, but only to provide prior exposure to basic ideas. This course rests on quantum theory in just a finite-dimensional setting, so the principal mathematical ingredients (from finite-dimensional linear algebra) will be readily accessible. We will begin by expounding the postulates of quantum mechanics in this setting (using Dirac notation) and then immediately make connections to information (quantum states viewed as information carriers, quantum teleportation) and computation (notion of qubits and quantum gates). Then we will discuss quantum cryptography (quantum key distribution), and quantum computing, culminating in an exposition of principal quantum algorithms, including the Deutsch–Jozsa algorithm, Grover's searching algorithm and an

overview of Shor's quantum factoring algorithm. The course is cross-disciplinary in its conceptual ingredients and will be of interest to pure and applied mathematicians alike.

4. Class/Module title *Topics in Analysis*

Tripos Mathematics

Part *IIC*

Term (Michaelmas or Lent) *Lent*

Number of lectures *24*

Lecture times, if available *T, Th, S 10am*

Caltech units *9*

Caltech evaluator *Oleg Ivrii*

Type of Caltech credit (option, general, etc.) *Option Elective Credit*

State CIT equivalent course, if applicable *Ma 111a*

Course description (paste in paragraph)

Some students find the basic courses in Analysis in the first two years difficult and unattractive. This is a pity because there are some delightful ideas and beautiful results to be found in relatively elementary Analysis. This course represents an opportunity to learn about some of these. There are no formal prerequisites: concepts from earlier courses will be explained again in detail when and where they are needed. Those who have not hitherto enjoyed Analysis should find this course an agreeable revelation.

Cambridge Proposed Course List – Michaelmas Term

Total CIT Units for term abroad: 36

Course by Correspondence/Units: 0

1. Class/Module title *Probability and Measure*

Tripos Mathematics

Part *IID*

Term (Michaelmas or Lent) *Michaelmas*

Number of lectures *24*

Lecture times, if available *M, W, F 11am*

Caltech units *9*

Caltech evaluator *Omer Tamuz*

Type of Caltech credit (option, general, etc.) *Option Elective Credit.*

State CIT equivalent course, if applicable *Ma 144a.*

Course description (paste in paragraph)

Measure theory is basic to some diverse branches of mathematics, from probability to partial differential equations. This course combines a systematic introduction to measure theory with an account of some of the main ideas in probability. You will be familiar with the Riemann integral from Parts IA and IB and have done some elementary probability in IA. The expectation operator of probability behaves somewhat like the integral, and in this course we see that they are both examples of some more general integral. These general integrals and the measures which

underlie them have advantages over the Riemann integral, even for functions defined on the reals. In Part IA the definition and properties of expectation were only partially explored and here we do it more fully. If you like to see how a substantial and coherent mathematical theory is put together, you will enjoy the measure theory part of this course, and this will be essential to any further work you do in analysis. It also underpins the probability which provides motivation and application throughout the course. The course ends with the Strong Law of Large Numbers and Central Limit Theorem, both of which are of real practical importance, being the mathematical basis for the whole of statistics. A good book to read for the early part of the course is *Probability with Martingales*, by D. Williams (CUP, 1991).

2. Class/Module title *Riemann Surfaces*

Tripos Mathematics

Part IID

Term (Michaelmas or Lent) *Michaelmas*

Number of lectures *16*

Lecture times, if available *T, Th 10am*

Caltech units *9*

Caltech evaluator Tom Graber

Type of Caltech credit (option, general, etc.) *Option Elective Credit.*

State CIT equivalent course, if applicable

Course description (paste in paragraph)

*A Riemann surface is the most general abstract surface on which one can define the notion of an analytic function, and hence study complex analysis. Roughly speaking, a surface is made into a Riemann surface when the change from one local coordinate system to another system is given by an analytic function. Not every Riemann surface has a global coordinate system; this accounts for both the interesting and the difficult parts of the theory. The course begins with a study of the Riemann sphere (which is just the complex plane with infinity attached) and of elliptic functions (that is to say, doubly periodic analytic functions) which are the analytic functions defined on a torus. Abstract Riemann surfaces and holomorphic maps are then introduced and some of the results already studied in earlier courses on complex analysis are extended to this more general context. Another view on Riemann surfaces comes from Riemann's original idea that the so-called 'multivalued functions' are just considered on a wrong domain: the natural domain is a surface covering the complex plane several (possibly infinitely many) times. This surface is called the Riemann surface of an analytic function and is obtained by the process of analytic continuation, extending the function (while keeping it analytic) in a maximal way from a domain in \mathbb{C} . The last part of the course shows that most Riemann surfaces carry their own intrinsic non-Euclidean geometry; thus complex analysis is much more closely connected to non-Euclidean geometry than to Euclidean geometry (despite the fact that it is first studied in the Euclidean plane). Prerequisite for this course is IB Complex Analysis (some knowledge of Analysis II will also be useful, especially for elliptic functions). Related Part II courses include those on Algebraic Topology, Algebraic Geometry and Differential Geometry. As a preliminary reading, consider the early parts of G.A. Jones and D. Singerman, *Complex functions* CUP, 1987, and of A.F. Beardon, *A primer on Riemann surfaces* CUP, 1984.*

3. Class/Module title *Classical Dynamics*

Tripes *Mathematics*

Part *IIC*

Term (Michaelmas or Lent) *Michaelmas*

Number of lectures *24*

Lecture times, if available *M, W, F 12pm*

Caltech units *9*

Caltech evaluator Tom Graber

Type of Caltech credit (option, general, etc.) *Option Elective Credit.*

State CIT equivalent course, if applicable *Ph 106a*

Course description (paste in paragraph)

This course follows on from the dynamics sections of Part IA Dynamics and Relativity and also uses the Euler–Lagrange equations from Part IB Variational Principles. The laws of motion for systems of particles and for rigid bodies are derived from a Lagrangian (giving Lagrange’s equations) and from a Hamiltonian (giving Hamilton’s equations) and are applied, for example, to the axisymmetric top. One advantage of the formalism is the use of generalised coordinates; it is much easier to find the kinetic and potential energy in coordinates adapted to the problem and then use Lagrange’s equations than to work out the equations of motion directly in the new coordinates. At a deeper level, the formalism gives rise to conserved quantities (generalisations of energy and angular momentum), and leads (via Poisson brackets) to a system which can be used as a basis for quantization. The material in this course will be of interest to anyone planning to specialise in the applied courses. It is not used directly in any of the courses but an understanding of the subject is fundamental to Theoretical Physics.

4. Class/Module title *Number Theory*

Tripes *Mathematics*

Part *IIC*

Term (Michaelmas or Lent) *Michaelmas*

Number of lectures *24*

Lecture times, if available *M, W, F 10am*

Caltech units *9*

Caltech evaluator Tom Graber

Type of Caltech credit (option, general, etc.) *Option Elective Credit.*

State CIT equivalent course, if applicable

Course description (paste in paragraph)

*Number Theory is one of the oldest subjects in mathematics and contains some of the most beautiful results. This course introduces some of these beautiful results, such as a proof of Gauss’s Law of Quadratic Reciprocity, and a proof that continued fractions give rise to excellent approximations by rational numbers. The new RSA public codes familiar from Part IA Numbers and Sets have created new interest in the subject of factorisation and primality testing. This course contains results old and new on the problems. On the whole, the methods used are developed from scratch. You can get a better idea of the flavour of the course by browsing Davenport *The Higher Arithmetic* CUP, Hardy and Wright *An introduction to the theory of numbers* (OUP, 1979) or the excellent *Elementary Number Theory* by G A and J M Jones. (Springer 1998).*